10.3: Polar Equations

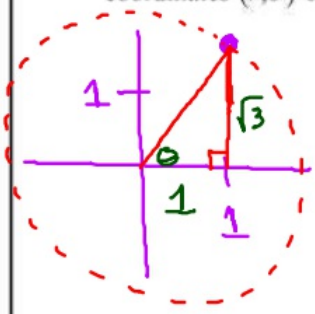
What you'll Learn About

- Calculus involving Polar Equations

1. Plot the rectangular (x, y) coordinate $(1, \sqrt{3})$. Then determine the polar coordinates (r, θ) of the point.

Rect → $(1, \sqrt{3})$

Polar → $(2, \pi/3)$



(r, θ) — r: radius, θ: angle

$r = \sqrt{1^2 + \sqrt{3}^2}$

$r = \sqrt{1+3}$

$r = \sqrt{4}$

$r = 2$

$\tan\theta = \frac{y}{x}$

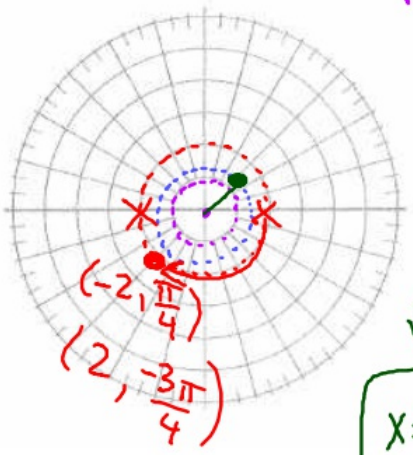
$\tan\theta = \frac{\sqrt{3}}{1}$

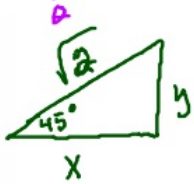
$\theta = \tan^{-1}(\sqrt{3})$ $\quad \theta = 60^\circ = \pi/3$

2. Plot the polar (r, θ) coordinate $\left(\sqrt{2}, \frac{\pi}{4}\right)$. Then determine the rectangular coordinate (x, y) of the point.

$(-2, \pi/4)$

$r = 2$





$\cos\frac{\pi}{4} = \frac{x}{\sqrt{2}}$ $\quad \sin\frac{\pi}{4} = \frac{y}{\sqrt{2}}$

$\sqrt{2}\cos\frac{\pi}{4} = x$ $\quad \sqrt{2}\sin\frac{\pi}{4} = y$

$x = r\cos\theta$ $\quad y = r\sin\theta$

If r and $\frac{dr}{d\theta}$ are the same sign the particle is moving away from the origin

If r and $\frac{dr}{d\theta}$ are opposite signs the particle is moving toward the origin

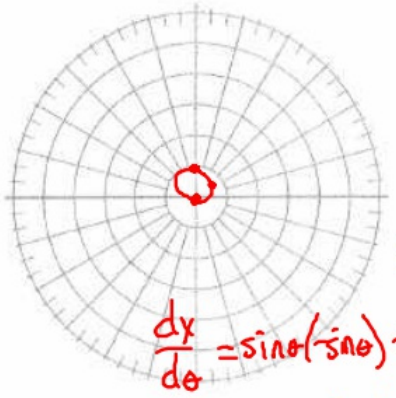
If $\frac{dr}{d\theta}=0$, the particle is the furthest from the origin

If r = 0 and $\frac{dr}{d\theta}$ is positive or negative, the particle is getting ready to move away from the origin

3. Graph the circle $r=\sin\theta$. Then determine the slope of the curve at $\theta=\frac{\pi}{4}$ and then determine $\frac{dr}{d\theta}$ and interpret what it means

dy/dx

r	θ
0	0
.707 = √2/2	π/4
1	π/2



$x = r\cos\theta$ $\quad y = r\sin\theta$

$x = \sin\theta\cos\theta$ $\quad y = (\sin\theta)^2$

$\frac{dx}{d\theta} = \sin\theta(-\sin\theta) + \cos\theta\cos\theta$ $\quad \frac{dy}{d\theta} = 2\sin\theta\cos\theta$

$= -\sin^2\theta + \cos^2\theta$

$\frac{dy}{dx} = \frac{2\left(\sqrt{2}/2\right)\left(\sqrt{2}/2\right)}{-\left(\sqrt{2}/2\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2} = \frac{}{0}$

UNDEFINED

4. Graph the circle $r=2\cos\theta$. Then determine the slope of the curve at $\theta=\frac{\pi}{4}$ and then determine $\frac{dr}{d\theta}$ and interpret what it means

r	θ
2	0